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DIFFERENCE MATRICES AND ORTHOGONAL ORTHOMORPHISMS OF GROUPS

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Abstract. Let G be a finite group of order v and let $k \geq 2$ be an integer. A (v, k, λ)-*difference matrix* (DM) over G, briefly (G, k, λ)-DM, is a $k \times \lambda v$ matrix $D = (d_{ij})$ with entries from G, such that for any two distinct rows x and y, the multiset of differences $\{d_{xj}^{-1} d_{yj} : 1 \leq j \leq \lambda v\}$ contains each element of G exactly λ times.

Difference matrices have a very close relationship with orthogonal orthomorphisms of groups. A bijection $\theta : G \to G$ of a finite group G is an *orthomorphism* of G if the mapping $x \mapsto x^{-1}\theta(x)$ is also a bijection, and two orthomorphisms θ and ϕ of G are said to be *orthogonal* if the mapping $x \mapsto \theta(x)^{-1}\phi(x)$ is a bijection. There exists a set of k pairwise orthogonal orthomorphisms of G if and only if there exists a $(G, k+2, 1)$-DM.

By giving previously unknown a pair of orthogonal orthomorphisms of cyclic groups of order $18t + 9$ for any positive integer t, we complete the existence spectrum of a pair of orthogonal orthomorphisms of cyclic groups. As a corollary, we complete the existence spectrum of a difference matrix with four rows over any finite abelian group.

Let H be a finite abelian group and let $D_{2H} = \langle H, b \mid b^2 = 1, bhb = h^{-1}, h \in H\rangle$ be the generalized dihedral group of H. It is proved that a $(D_{2H}, 4, 1)$-DM exists if and only if H is of even order and H is not isomorphic to $\mathbb{Z}_4$. It is proved that if G is a finite abelian group and the Sylow 2-subgroup of G is trivial or noncyclic, then a $(G, 5, 1)$-DM exists, except for $G \in \{\mathbb{Z}_3, \mathbb{Z}_2 \oplus \mathbb{Z}_2, \mathbb{Z}_4 \oplus \mathbb{Z}_2, \mathbb{Z}_9\}$ and possibly for some groups whose Sylow 2-subgroup lies in $\{\mathbb{Z}_2 \oplus \mathbb{Z}_2, \mathbb{Z}_4 \oplus \mathbb{Z}_2, \mathbb{Z}_{32} \oplus \mathbb{Z}_2, \mathbb{Z}_{16} \oplus \mathbb{Z}_4\}$, and some cyclic groups of order $9p$ with p prime.

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