On a problem related to centralizers in groups

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Let G be a group and let x be an element of G. Denote by $C_G(x)$ the centralizer of x in G. Then clearly $\langle x\rangle \leq C_G(x)$.

An element x of $G \setminus \{1\}$ will be called *deficient* if

$$\langle x\rangle < C_G(x),$$

non-deficient if

$$\langle x\rangle = C_G(x).$$

If $x \in G$ is deficient (non-deficient), then the conjugacy class x^G of x in G will be also called *deficient* (*non-deficient*).

Let j be a *non-negative* integer. We will say that the group G has *defect* j, denoted by $G \in D(j)$ or by the phrase "G is a $D(j)$-group", if **exactly** j non-trivial conjugacy classes of G are deficient.

In this talk we report some results on arbitrary finite or infinite $D(0)$-groups and $D(1)$-groups, obtained in [3].

Then, more generally, we shall deal with the class $M(j)$ of $D(j)$-groups G, $j \geq 1$, satisfying the following condition: G contains an element x of order p^{j+1}, for some prime p (see [5]).

Notice that if G is a $D(0)$-group, or G is a periodic $D(1)$-group, or G is in the class $M(j)$, then every element of G has order a power of a prime. We also report some known results on finite and locally finite groups with this property (see [1], and [2]), and some new results about some classes of infinite groups with this property (see [4]).

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References

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