ON SOME OPEN PROBLEMS IN ALGEBRAIC GRAPH THEORY

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ABSTRACT. Out of many long standing open problems in algebraic graph theory (AGT) and permutation groups (PG), two are especially dear to me. First, the question asked by Lovász in 1970 tying together traversability and symmetry, two seemingly unrelated graph-theoretic concepts: *Does every finite connected vertex-transitive graph have a Hamilton path?* And second, a long standing problem regarding existence of nontrivial automorphisms with all orbits of the same size (the so called *semiregular* automorphisms) in vertex-transitive graphs, these days going by the name *The polycirculant conjecture* [5].

In this talk I will discuss some recent results regarding constructions of Hamilton cycles admitting rotational symmetry, a concept effectively tying together the above two problems. Moreover, time permitting, I will also touch upon the role of the classification of finite simple groups (CFSG) and the importance of CFSG-free approaches to certain problems in AGT and PG [1].

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