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ON THE SPLITTING FIELD OF CAYLEY (DI)GRAPHS

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Abstract. A (di)graph Γ is called a Cayley (di)graph over a group G if its automorphism group admits a regular subgroup, acting on the vertex set, isomorphic to G. It is well-know that one can compute the eigenvalues of the characteristic polynomial of the adjacency matrix of a Cayley (di)graph over a group G using irreducible complex representations of G. Furthermore, every eigenvalue of a (di)graph Γ is an algebraic integer in some algebraic extension K of the rational field $\mathbb{Q}$, where K is called the splitting field of Γ. The algebraic degree of a Γ, $\deg(\Gamma)$ is the defined as the extension degree $[K : \mathbb{Q}]$. Cayley (di)graphs with splitting field $\mathbb{Q}$ are called integral. These Cayley (di)graphs are studied extensively. In this talk, we determine the splitting field of Cayley (di)graphs over some special groups. Also we characterize all finite abelian groups having a connected Cayley graph with algebraic degree 2 and with valency up to 5. This talk is based on my recent joint works with A. Abdollahi, M. Ebrahimi, T. Feng and S. Wang.

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