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THE SECOND LARGEST EIGENVALUE OF CAYLEY GRAPHS

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Abstract. The eigenvalues of a graph are defined as the eigenvalues of its adjacency matrix, and the Laplacian eigenvalues of a graph are the eigenvalues of its Laplacian matrix. It is well-known that the second largest eigenvalue of Cayley graphs plays an important role in many applications especially in the context of expander graphs. Aldous conjectured that for any set T of transpositions in S_n, the Cayley graph $\mathrm{Cay}(S_n, T)$ has the same algebraic connectivity as the graph with vertex set $[n]$ and edges $\{i, j\}$ for $(i, j) \in T$, where the algebraic connectivity of a graph is its second smallest Laplacian eigenvalue (in particular, the algebraic connectivity of a Cayley graph is the difference between its degree and second largest eigenvalue). This conjecture in its general form was proved by Caputo, Liggett and Richthammer in 2010. In this talk I will review some results on the second largest eigenvalue of Cayley graphs with a focus on Aldous' conjecture and its generalizations. The talk includes joint work with Yuxuan Li and Binzhou Xia.

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